Equilibrium Selection
as a matter of norms and beliefs

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Game Theory and observed behavior
Is there a connection?

➢ „Naive“ applications reveal fundamental differences!
➢ Is there any connection at all? Analytical Sociology: No!
➢ Amendments (wide psychological version)
  • Norms (social preferences) instead of egoism
  • Beliefs instead of complete information
  • Error or imprecision are sometimes rather successful!

E.g. Quantal response equilibria (McKelvey&Palfrey, 1995) with social preferences
Additional Complication(?)

Multiple Equilibria

- 2x2 games often have three equilibria
- The 4x2 games discussed below have up to 31 equilibria
- Can players coordinate on one of the equilibria?
  - If yes: Which one is played?
  - If no: ?
Normative approaches to equilibrium selection

- Pay-off dominance (if applicable)
- Risk dominance (different definitions)
- Global games (noise → 0)
- Quantal response equilibria (impresision → 0)
- Harsanyi-Selten theory
- ....

Always – often – sometimes: unique selection

Is „unique“ desirable for a behavioral approach?
Behavioral Theory of Equilibrium Section

Non-existent (?)

Requirements?
General Hypothesis

Behavior is based on three main requirements:
• Consistency (best replies, equilibria)
• Efficiency (social product maximizing strategies)
• Fairness (qualitative or quantitative equality)

However, people are prone to
• Error

as random deviations and non-justified beliefs.

Evidence for each of these behavioral traits from economic experiments!
Specific Hypothesis

Behavior is an **equilibrium strategy** either from

- the **most efficient** equilibrium
  
or

- the **most efficient among the fair** equilibria

[Fairness= binary concept : Equilibria are either fair or unfair]

But ....
Plus Error!

Concerning

- Equilibrium (non-equilibrium heuristics)
- Maximum (second best)
- Implementation (probability of deviation)
Practical Hypothesis

Players belong to different populations

- PE1 play most efficient equilibrium
- PE2 play second most efficient equilibrium
- PF1 play most efficient among the fair equilibria
- PF2 play second most efficient among the fair equilibria
- ... use simple heuristics

In addition:
Small random deviations from all strategies
The Practical Hypothesis defines a **strict frame** with some **degrees of freedom**, in particular concerning

- Definition of fairness
- Heuristics
Experiments:
- Binary Threshold Public Good games
- 4 players
- 2 strategies (contribute with costs = ci or not with costs = 0)
- Public good produced if \( \geq k \) players contribute
  Public good provides benefits \( G_i \), otherwise 0

In the positive frame:
\( k=1 \) is the Volunteer‘s Dilemma (Diekmann, 1985)
\( k=4 \) is the Stag Hunt Game (Rousseau, 1762)
Experimental design
- 4 treatments x 4 games
- Games with k=1,2,3,4
- Treatments S+, S-, A, B
  In S+ two kinds of players with positive ci and Gi and ci/Gi=0.4
  In S- all players as in S+ but with negative ci and Gi
  In A all players with positive costs and benefits and cost/benefit ratios = (0.225, 0.25, 0.275, 0.3)
  In B all players with positive costs and benefits and cost/benefit ratios = (0.1, 0.2, 0.3, 0.4)
Experimental design

- Sessions with 8 players (two games with 4 players)
- In every session 4x8 periods (repetitions of games)
- Same k in 8 consecutive periods, random order of k
- Stranger design (in every period random allocation)

- S+, S- with 10 sessions each in Frankfurt/Oder
- A with 6 (12) sessions in Frankfurt (Berlin)
- B with 10 (6) sessions in Frankfurt (Berlin)
## Number of equilibria

<table>
<thead>
<tr>
<th>Threshold k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># pure str. equ.</td>
<td>4</td>
<td>7</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td># compl. mixed equ.</td>
<td>≤1</td>
<td>≤2*</td>
<td>≤2*</td>
<td>≤1</td>
</tr>
<tr>
<td># pure/mixed equ.</td>
<td>≤10</td>
<td>≤24</td>
<td>≤24</td>
<td>≤6</td>
</tr>
</tbody>
</table>
Hypothetical populations

- PE1 play most efficient equilibrium
- PE2 play second most efficient equilibrium
- PF1 play most efficient among the fair equilibria
- PF2 play second most efficient among the fair equilibria
- P1 contribute always (always fair, equ.* for k=4)
- P0 contribute never (always fair, equ.* for k=2,3,4)
These do not seem to be binomial distributions!
No unique equilibrium selection!
Parameters to be estimated

- Population shares for P1, PE1, PE2, PF1, PF2, P0
- Warm glow parameters varying with cost/benefit ratios ci/Gi
- One deviation probability

- 7 Parameters in S+ and S-
- 10 parameters in A and B
<table>
<thead>
<tr>
<th>Data</th>
<th>N</th>
<th>Minimum $\chi^2$</th>
<th>Minimum $\chi_r^2$</th>
<th>Maximum Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\chi^2$</td>
<td>$p(\chi^2)$</td>
<td>-$\log L$</td>
</tr>
<tr>
<td>S+/S- per&lt;17</td>
<td>320</td>
<td>171.0</td>
<td>0.002</td>
<td>712.1</td>
</tr>
<tr>
<td>S+/S- per&gt;16</td>
<td>320</td>
<td>146.1</td>
<td><strong>0.060</strong></td>
<td>602.9</td>
</tr>
<tr>
<td>S+/S- all</td>
<td>640</td>
<td>190.8</td>
<td>&lt;10^{-4}</td>
<td>1342.5</td>
</tr>
<tr>
<td>A_{TU}</td>
<td>384</td>
<td>121.0</td>
<td><strong>0.405</strong></td>
<td>610.5</td>
</tr>
<tr>
<td>A_{V}</td>
<td>192</td>
<td>141.9</td>
<td><strong>0.066</strong></td>
<td>350.9</td>
</tr>
<tr>
<td>A_{TU}+ A_{V}</td>
<td>576</td>
<td>181.7</td>
<td>10^{-4}</td>
<td>986.7</td>
</tr>
<tr>
<td>B_{TU}</td>
<td>192</td>
<td>124.2</td>
<td><strong>0.300</strong></td>
<td>291.0</td>
</tr>
<tr>
<td>B_{V}</td>
<td>320</td>
<td>122.0</td>
<td><strong>0.382</strong></td>
<td>549.3</td>
</tr>
<tr>
<td>B_{TU}+ B_{V}</td>
<td>512</td>
<td>135.5</td>
<td><strong>0.129</strong></td>
<td>841.3</td>
</tr>
</tbody>
</table>

Table 4: Minimum Chi-square and Maximum likelihood estimation of the finite mixture model with six data sets under HypThresh.
Estimated population shares (%)
Estimated warm glow parameters
(additional utility from contributing)
Performance of Equ. Select. hypothesis where applicable (static behavior, same subject pool)

- Not rejected in chi-square tests
- Same population shares for $k=1,2,3,4$ (and $S+/S-$)
- Warm glow parameters varying only with $c_i/G_i$

But remaining treatment effect:
- Different population shares in $S+/S-$, A, and B
Open questions

- Explanation of remaining treatment effects
- Application to other classes of games
- Populations and personal characteristics
- Extension to dynamic behavior (learning)

Thank you for your attention!
In spite of the good fit, ....

Fundamental problem in repeated games: Why stick to equilibria which are not played by all others? Possible answers:

- People have detected the „right thing“ and they stick to it, independent of what others do (Cooper, 1996, rep. PD, 12% always coop.)
- There is no advantage from changing one‘s strategy
- Deviationed from mixed strategy equilibria are difficult to detect